MATH 163 **Name:_____Solution Key – Answers only**

Final Exam Practice

1. Solve the following equations:

a. $x = 34$

b $x = \{\pm 2i\sqrt{2}, \pm 1\}$

c $x = \{11, -7\}$

d. $x = 9998$

e. $\frac{(log_6 10)-1}{-2} = x$ $x \approx -0.1425$

f. $x = \frac{8}{3}\left(e^{10} - 1\right)$

g. Since you can't take the absolute value of a number and get a negative answer, there is NO SOLUTION. Remember: Absolute value is the distance a number is from zero.

h. $x = 10$ (*solution*); $x \neq 5$ (*extraneous solution*)

i. $x = \dfrac{\dfrac{2}{\log_5 6} + 1}{2} \approx 1.398$

j. $x = 6,\ -1,\ 2,\ or\ 3$

2. $x \geq 5$ $x \leq -\frac{1}{2}$

$\left(-\infty, -\frac{1}{2}\right] \cup [5, \infty)$

b. $\left(-\infty, -\frac{5}{3}\right] \cup [4, \infty)$

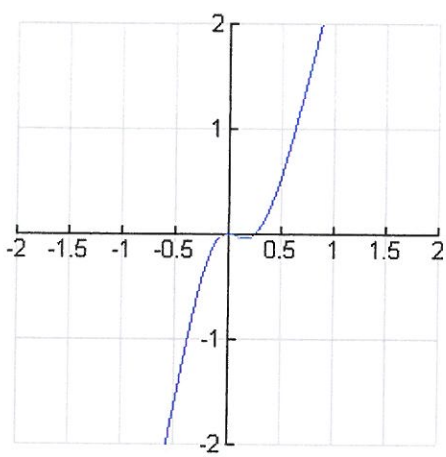
c. $\frac{-4}{3} < x < \frac{2}{3}$

$\left(-\frac{4}{3}, \frac{2}{3}\right)$

d $[-5, -3) \cup [0, 3)$.

3. determine the:

Function	a. Domain	b. X-Int(s)	c. Vert. Asymp(s)	d. Horiz. Asymp	e. Oblique/Slant Asymp
f(x)	$(-\infty,-2)\cup(-2,2)\cup(2,\infty)$	None	$x=2$ $x=-2$	$y=\frac{5}{2}$	None
g(x)	$(-\infty,\infty)$	$x=0$ $x=\frac{1}{4}$	None	None	$y=4x-1$
h(x)	$(-\infty,-1)\cup(-1,4)$ $\cup(4,\infty)$	None	$x=-1$	$y=0$	None



_____

4. Given $f(x)=\frac{3x}{2x+5}$; $h(x)=x^2-3x-28$; $g(x)=\sqrt{\frac{4}{3}x-1}-9$; $p(x)=6x-3$

a. State the domain of f(x)

$\left(-\infty,\frac{-5}{2}\right)\cup\left(-\frac{5}{2},\infty\right)$

b. State the domain of g(x)

$\left[\frac{3}{4},\infty\right)$

c. Determine f(x + 2)

d. Determine g(3x)

$$f(x)=\frac{3(x+2)}{2(x+2)+5}$$
$$=\frac{3x+6}{2x+9}$$

$$g(x)=\sqrt{\frac{4}{3}(3x)-1}\ -9$$
$$=\sqrt{4x-1}\ -9$$

e. Determine (f ○ g)(3/4) =

$$=\frac{27}{13}$$

f. Determine (g ○ p)(x)

$$=\sqrt{8x-5}-9$$

g. Determine $f^{-1}(x)$

$$f^{-1}(x)=\frac{-5x}{2x-3}$$

h. Determine $g^{-1}(x)$

$$g^{-1}(x)=\frac{3}{4}\left[(x+9)^2+1\right],\quad x\geq -9$$

i. Determine (p - h) (x)

$$=-x^2+9x+25$$

j. Determine (h+ p)(x)

$$=x^2+3x-31$$

k. (hp)(x)

$$=6x^3-21x^2-159x-3x^2+84$$

l. The domain of $\left(\frac{p}{h}\right)(x)$

$$\frac{6x-3}{(x-7)(x+4)};\ x\neq 7,-4$$

$$(-\infty,-4)\cup(-4,7)\cup(7,\infty)$$

5. Show that $f(x)=2x+7$ and $g(x)=\frac{x-7}{2}$ are inverses of each other.

Need to prove that $(f \circ g)(x) = (g \circ f)(x) = x$.

$$(f \circ g)(x) = f[g(x)] = f\left[\frac{x-7}{2}\right]$$
$$= 2\left(\frac{x-7}{2}\right) + 7$$
$$= x - 7 + 7$$
$$= x$$

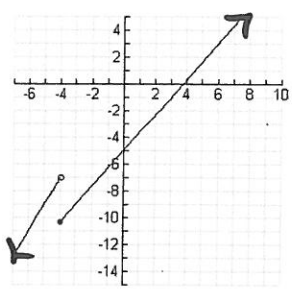
$$(g \circ f)(x) = g[f(x)] = g[2x+7]$$
$$= \frac{2x+7-7}{2}$$
$$= \frac{2x}{2} = x$$

6. Given $f(x) = \begin{cases} 2x+1 & x < -4 \\ \frac{4}{3}x - 5 & x \geq -4 \end{cases}$

a. Evaluate f(9) =7

b. Evaluate f(-7)= -13

c. graph f(x).



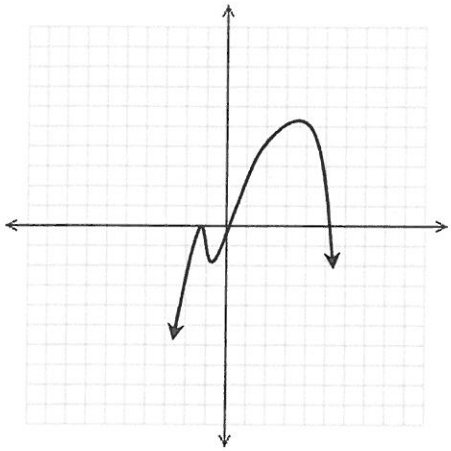
7. For the polynomials given below, **list each real zero and its multiplicity**. Then draw a graph of the polynomial.

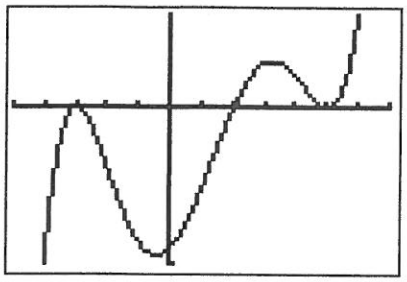
a. $f(x) = -9x(x-5)(3x+5)^2$

x = 0 multi. 1
x = 5 multi. 1
x = $-\frac{5}{3}$ multi. 2

b. $g(x) = 4(x+3)^2(x-2)(x-5)^2$

x = -3 multi. 2
x = 2 multi. 1
x = 5 multi. 2





8. Given polynomial $g(x) = x^4 + 4x^3 + 2x^2 - x + 6$,
   a. What is the maximum # of zeroes g(x) could have? Four
   b. What is the maximum # of turning points g(x) could have? Three
   c. list all of the potential rational zeroes of g(x) $x = \pm 1, \pm 2, \pm 3, \pm 6$
   d. Is $(x - 3)$ a factor of g(x)? Explain.
   No, (x-3) is not a factor of g(3).
   e. Write g(x) in its fully factored form. $(x + 2)(x + 3)(x^2 - x + 1)$
   f. Find all zeroes (real and complex) of g(x)

$$x = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

So, the real zeros are $x = -3$ and $x = -2$, and the imaginary zeros are $x = \frac{1 \pm i\sqrt{3}}{2}$.

9. Find all of the zeroes (real and complex) of the polynomials below:

   a. $f(x) = 3x^3 - x^2 + 27x - 9$

$$x = \left\{ \frac{1}{3}, \pm 3i \right\}$$

b. $g(x) = (x + 3)(x^2 - 4x - 1)$

$$x = \left\{ -3, \frac{7 \pm 2\sqrt{5}}{2} \right\}$$

10. Form a polynomial with real coefficients of degree 4 that has zeroes: $x = 0$, $x = -1$, $x = -5i$

$$= x^4 + x^3 + 25x^2 + 25x$$

11. Suppose the function $h(t) = -16t^2 + 50t + 15$ gives the height of an object in feet over time (in seconds).

a. When does the object reach its highest point?

$$= \frac{25}{16} \text{ seconds}$$

b. When does the object hit the ground?

$$= \frac{-50 \pm 2\sqrt{865}}{-32}$$
$$= 3.4, -0.28$$

The ball will hit the ground after 3.4 seconds.

12. Suppose a company finds that the revenue, in dollars, from sales of a particular product is a function of the unit price **p**, in dollars, that it charges. The revenue **R** is given by:

$$R(p) = -\frac{1}{4}p^2 + 1500p$$

a. What unit price should be set to maximize revenue?
$= 3000$ Dollars

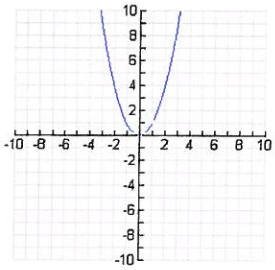
b. What is the maximum revenue?

$2,250,000.

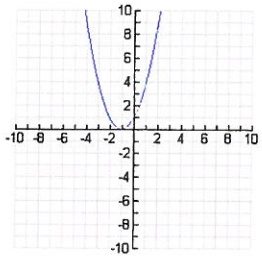
13. Sketch the graph of the following using transformations:

a. $f(x) = -(x+1)^2 - 2$

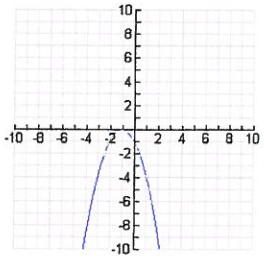
Step 1



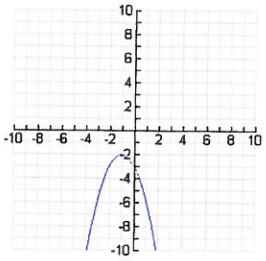
Step 2



Step 3

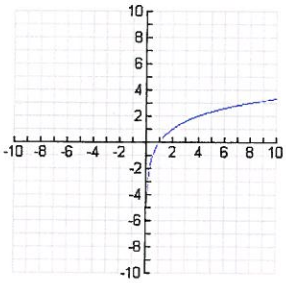


$f(x) = -(x + 1)^2 - 2$

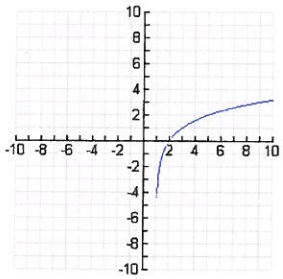


b. $g(x) = \log_2(x - 1) + 3$

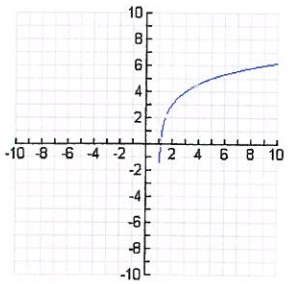
Step 1



Step 2

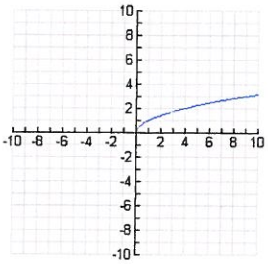


$g(x) = \log_2(x - 1) + 3$

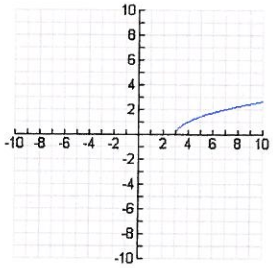


c. $f(x) = 2\sqrt{x - 3}$

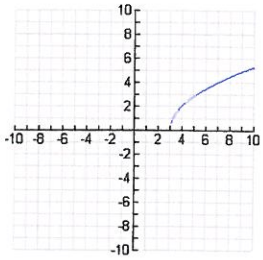
Step 1



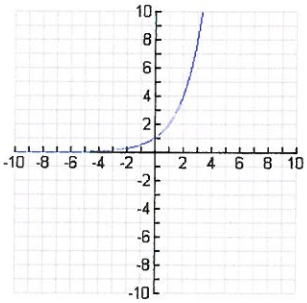
Step 2

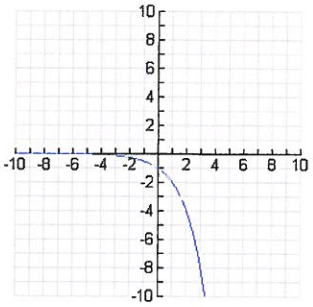


$f(x) = 2\sqrt{x - 3}$

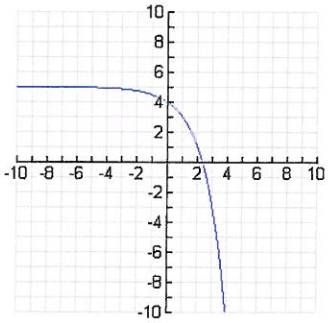


d. $h(x) = -2^x + 5$ Step 1 Step 2

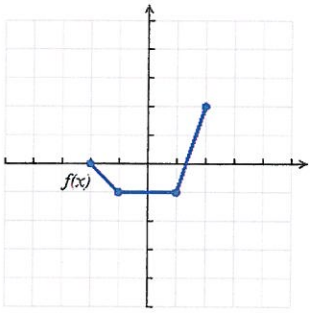


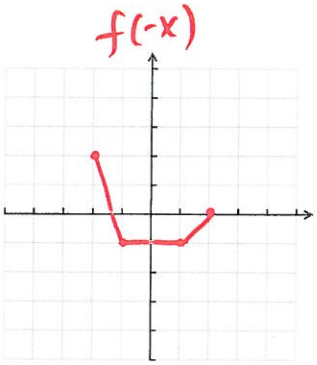


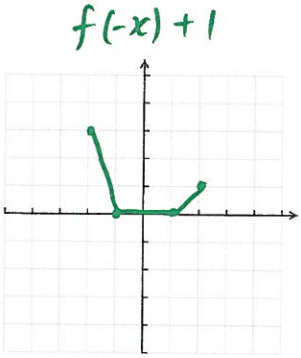
$h(x) = -2^x + 5$



e. $f(-x)+1$

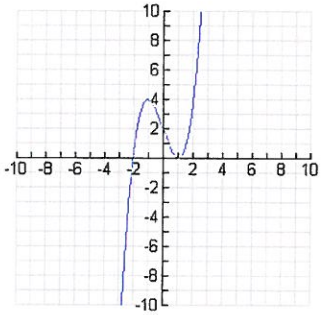


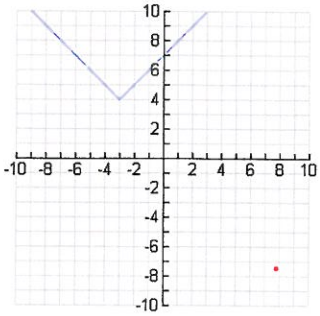




14. Write a function for each graph below that could describe it:

a.





$(x + 2)(x - 1)^2$ $|x + 3| + 4$

15. Identify the transformations involved to get from $f(x) = \sqrt{x}$ to

a. $g(x) = -\sqrt{x - 4}$ b. $h(x) = 3\sqrt{x} + 5$

a. H shift right 4, then reflect over the x-axis

b. vertical stretch by a factor of 3 and then up 5

16. (6 points) Given that $-2i$ is a zero of $P(x) = x^4 + 3x^3 + 12x - 16$ find the remaining zeros.

$zeros: 2i, -2i, -4, 1$

17. The decay model $A(t) = 34e^{-.00244t}$ describes the amount of Strontium 90 left in a sample that is present after t years, if 34 grams are present to begin with.

a. How much Strontium 90 will be left after 100 years?
≈ 26.64 grams
b. How long will it take until 20 grams remain?
Given: $A(t) = 20$

$$\frac{\ln \frac{10}{17}}{-0.00244} = t$$

$$t \approx 217.47 yrs$$

18. The growth model $P(t) = 300e^{0.03t}$ describes the size P of an insect population after t days, if 300 insects are present to begin with.

a. What is the population after 8 days?

≈ 381 insects

b. When will the insect population triple?

$$t \approx 36.62 days$$

19. Evaluate the following:

a) =2 b) =0 c) =-2 d) $= \frac{3}{2}$

20. Use either $A = P(1+\frac{r}{n})^{nt}$ *or* $A = Pe^{rt}$ to answer each of the following:

a) What amount results from a $480 investment at 7% compounded quarterly after 2 years?
$\approx \$551.46$

b) What amount results from a $12,000 investment at 5.7% compounded continuously after 8 years?
$\approx \$18,933.00$

c) How much should be invested to get $2000 after 6 years at 6% compounded semiannually?
$\approx \$1402.76$

d) How long does it take $1700 to double if it is invested at 5% interest compounded continuously?

$$t = \frac{\ln 2}{.05} \approx 13.9 years$$

21. Find the equation of the line that is parallel to 3x+2y=4 that passes through the point (2,5).

$$y = -\frac{3}{2}x + 8$$

22. Find the equation of the line that is perpendicular to $y = \frac{3}{5}x - 4$ that passes though the point (-1,3).

$$y = -\frac{5}{3}x + \frac{4}{3}$$

23. For each of the following, calculate the difference quotient: $\dfrac{f(a+h) - f(a)}{h}$

a) $= -5$

b) $= 6a + 3h - 2$

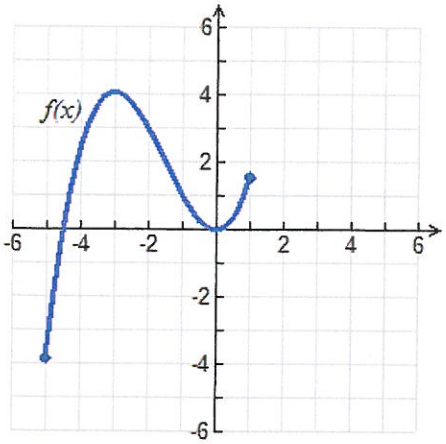
24. Use the binomial theorem to expand: a. $(x+1)^5$ b. $(2x+3)^4$

a. $= x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$

$= 16x^4 + 96x^3 + 216x^2 + 216x + 81$

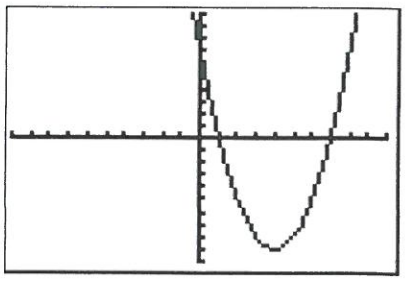
b.

25. Use the graph of the function $f(x)$ below to answer the following:
    a. Domain: $[-5,1]$
    b. Range: $[-4,4]$
    c. On which interval(s) of x is $f(x)$ decreasing? $(-3,0)$
    d. On which interval(s) of x *is* $f(x)$ increasing? $(-5,-3) and (0,1)$
    e. Determine a local minimum. 0 or $(0,0)$
    f. Determine a local maximum. 4 or $(-3,4)$



26. Graph each quadratic function by correctly finding its vertex and its x-intercepts and y-intercept.

a)

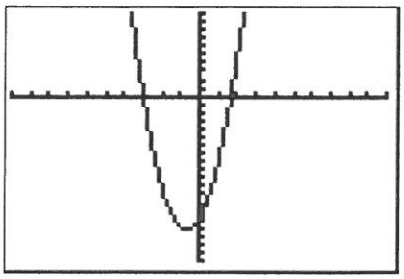


$Vertex: (4,-9)$

$X - Intercepts: (7,0),(1,0)$

$Y - Intercept: (0,7)$

b)



$Vertex: (-\frac{2}{3}, -\frac{49}{3})$

$X - Intercepts: (-3, 0), (\frac{5}{3}, 0)$

$Y - Intercept: (0, -15)$

27. Find the 3rd term in the binomial expansion of

$= 720x^3$

28. Determine whether the function is one-to-one. If so, state the inverse function.

a)

$One - to - one$

$f^{-1}(x) = \dfrac{x-8}{3}$

b)

$Not\ one - to - one$

c)

$one - to - one$

$h^{-1}(x) = \sqrt[3]{x-2}$