Practice Final Exam (Answers) Math 165

① $5^{x-2} = 30$

$\log_5 30 = x - 2$

$\frac{\log 30}{\log 5} + 2 = x$

$x \approx 4.11$

② $4\log_2(x) + 3 = 131$

$4\log_2(x) = 128$

$\log_2(x) = 32$

$x = 2^{32}$

$x = 4{,}294{,}967{,}296$

③ $\ln(2x+3) - \ln(3x-1) = 8$

$\ln\left(\frac{2x+3}{3x-1}\right) = 8$

$\frac{2x+3}{3x-1} = e^8$

$2x+3 = e^8(3x-1)$

$2x+3 = 3e^8x - e^8$

$2x - 3e^8x = -e^8 - 3$

$x(2-3e^8) = -e^8 - 3$

$x = \frac{-e^8-3}{2-3e^8}$

$x \approx .33$

④ $\log(x+4) - \log(x-5) = \log 52$

$\log\left(\frac{x+4}{x-5}\right) = \log 52$

$\frac{x+4}{x-5} = 52$

$x+4 = 52(x-5)$

$x+4 = 52x - 260$

$264 = 51x$

$\frac{264}{51} = x$

$x = \frac{88}{17}$

⑤ $3\log x + (1/3)\log y - 2\log z$

$= \log x^3 + \log \sqrt[3]{y} - \log z^2$

$= \log \dfrac{x^3\sqrt[3]{y}}{z^2}$

⑥ $\ln 5 + \ln x^2 + \ln y^4 - \ln 3 - \ln z$

$= \ln 5 + 2\ln x + 4\ln y - \ln 3 - \ln z$

⑦ Domain of $f(x) = \log(3x-12)$

$3x - 12 > 0$ Domain: $(4, \infty)$

$3x > 12$

$x > 4$

⑧ $\dfrac{3x-4}{(x-8)(x+4)} = \dfrac{A}{x-8} + \dfrac{B}{x+4} = \dfrac{5}{3(x-8)} + \dfrac{4}{3(x+4)}$

MULTIPLY BY DENOMINATOR

$3x - 4 = A(x+4) + B(x-8)$

$x = -4$ $\quad -16 = -12B$ $\qquad x = 8$ $\quad 20 = 12A$

$\dfrac{4}{3} = B$ $\qquad \dfrac{5}{3} = A$

(9) (Not on Final)

$$
\begin{array}{r|l}
 & x - 6 \\
\hline
x^2+6x+8 & x^3+0x^2+4x-2 \\
 & -(x^3+6x^2+8x) \\
\hline
 & -6x^2-4x-2 \\
 & -(-6x^2-36x-48) \\
\hline
 & 32x+46 \text{ (remainder)}
\end{array}
$$

$$\frac{x^3+4x-2}{x^2+6x+8} = x-6+\frac{32x+46}{x^2+6x+8}$$

$$\frac{32x+46}{x^2+6x+8} = \frac{32x+46}{(x+2)(x+4)} = \frac{A}{(x+2)} + \frac{B}{(x+4)} = \frac{-9}{x+2} + \frac{41}{x+4}$$

$$32x+46 = A(x+4) + B(x+2)$$

$x=-4$

$-82 = -2B$

$41 = B$

$x=-2$

$-18 = 2A$

$-9 = A$

$$\frac{x^3+4x-2}{x^2+6x+8} = x-6-\frac{9}{x+2}+\frac{41}{x+4}$$

(10) $(x-h)^2 + (y-k)^2 = r^2$ $\quad d=12$

$* \; (x-6)^2 + (y+5)^2 = 36$ $\quad r=6$

(11) $y = a(x-h)^2 + k$ $\quad (h,k) \rightarrow (2,-6)$

$(x,y) \rightarrow (-1,4)$

$4 = a(-1-2)^2 + -6$

$4 = 9a - 6$

$10 = 9a$ $\quad * \; y = \frac{10}{9}(x-2)^2 - 6$

$\frac{10}{9} = a$

(12) $a_1 = -23$

$d = 5$ $\quad a_{71} = -23 + (71-1)(5)$

$* \; a_{71} = 327$

(13) $a_8 = 24$ $\quad d = \frac{24 - -6}{8-3} = \frac{30}{5} = 6$

$a_3 = -6$ $\quad a_1 = -6 - (6)(2)$

$a_1 = -18$

$a_{44} = -18 + (44-1)(6)$

$* \; a_{44} = 240$

(14) $\sum_{n=1}^{22} (2n-3) = n\left(\frac{t_1+t_{22}}{2}\right)$

$t_1 = 2(1)-3 = -1$

$t_{22} = 2(22)-3 = 41$

$S_{22} = 22\left(\frac{-1+41}{2}\right)$

$* \; S_{22} = 440$

(15) $a^2+20^2 = 30^2$

$a^2+400 = 900$

$a^2 = 500$

$a \approx 22.36$

$\cos A = \frac{20}{30}$

$A = \cos^{-1}(20/30)$

$A \approx 48.2°$

$B \approx 41.8°$

(16) $m\angle B = 56°$

$\cos 34° = \frac{18}{c}$

$c \cdot \cos 34° = 18$

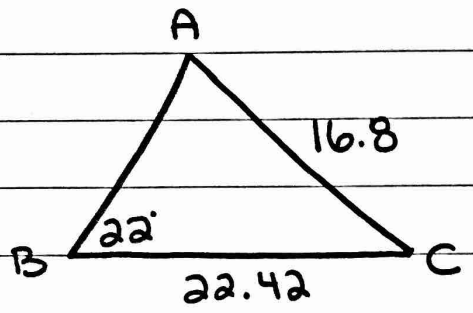
$c = \frac{18}{\cos 34°}$

$c \approx 21.71$

$\tan 34° = \frac{a}{18}$

$a \approx 12.14$

(17)



Triangle #1

$$\frac{16.8}{\sin 22^\circ} = \frac{22.42}{\sin A}$$

$\sin A \approx .4999214$

$A \approx 29.99 \approx 30^\circ$

$C \approx 128^\circ$

$$\frac{c}{\sin 128^\circ} = \frac{22.42}{\sin 30^\circ}$$

$c \approx 35.3$

Since $\measuredangle A \approx 30^\circ$, then $\measuredangle A$ could also have a measure of 150° so there are two triangles.

<u>Triangle #2</u>

$m\measuredangle A = 150^\circ$

$m\measuredangle C = 8^\circ$

$$\frac{c}{\sin 8^\circ} = \frac{16.8}{\sin 22^\circ}$$

$c \approx 6.24$

(18) $b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 50^2 + 55^2 - 2(50)(55) \cos 61^\circ$

$b^2 \approx 2858.547$

$b \approx 53.47$

$$\frac{\sin A}{50} = \frac{\sin 61^\circ}{53.47}$$

$m\measuredangle C \approx 64.13^\circ$

$\sin A \approx .81786$

$A \approx 54.87^\circ$

(19) $\frac{(\sin x)}{(\sin x)}\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\frac{(\cos x)}{(\cos x)} = \frac{1}{\sin x \cos x}$

$$\frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x} \checkmark$$

(20) $\frac{1 + \sec x}{\tan x + \sin x} = \boxed{\csc x}$

$$\frac{\left(1 + \frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x} + \sin x\right)} = \frac{\left(\frac{\cos x}{\cos x} + \frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x} + \frac{\sin x \cos x}{\cos x}\right)}$$

$$= \left(\frac{1 + \cos x}{\cancel{\cos x}}\right)\left(\frac{\cancel{\cos x}}{\sin x + \sin x \cos x}\right)$$

$$= \frac{\cancel{(1 + \cos x)}}{\sin x \cancel{(1 + \cos x)}} = \frac{1}{\sin x} = \boxed{\csc x} \checkmark$$

(21) $-\frac{\pi}{4} + 2\pi = \frac{7\pi}{4}$

$-\frac{\pi}{4} - 2\pi = \frac{-9\pi}{4}$