Name: KEY Math 165 Test #2

① Find the 85th term in an arithmetic sequence where $a_1 = -20$ and $a_8 = 1$.

$t_{85} = -20 + (85-1)(3)$ $\quad d = \frac{1 - -20}{8-1} = \frac{21}{7} = 3$

$t_{85} = -20 + (84)(3)$

$\boxed{t_{85} = 232}$

② What is the sum of the first 28 terms in an arithmetic sequence where $t_n = 5 + 2(n-1)$?

$t_1 = 5 + 2(1-1)$ $\quad t_{28} = 5 + 2(28-1)$

$t_1 = 5$ $\quad t_{28} = 59$ $\quad S_{28} = 28\left(\frac{5+59}{2}\right)$

$\boxed{S_{28} = 896}$

③ What is the first term of an arithmetic sequence where $t_{13} = 128$ and the common difference is 8?

$t_1 = 128 - (12)(8)$

$\boxed{t_1 = 32}$

④ Evaluate: $\sum_{n=1}^{18}(2n-5)$ $\quad t_1 = -3$ $\quad t_{18} = 31$

$S_{18} = 18\left(\frac{-3+31}{2}\right) = \boxed{252}$

⑤ Evaluate $\sum_{x=1}^{25} 2x^2 - 4x + 1$

$$2\sum_{x=1}^{25} x^2 - 4\sum_{x=1}^{25} x + \sum_{x=1}^{25} 1$$

$$= 2\left[\frac{25(26)(51)}{6}\right] - 4\left[\frac{25(26)}{2}\right] + 25$$

$$= 11{,}050 - 1{,}300 + 25 = \boxed{9{,}775}$$

⑥ Evaluate each expression: (must give exact values)

a) $\sin 60^\circ = \boxed{\frac{\sqrt{3}}{2}}$

b) $\cos 120^\circ = \boxed{-\frac{1}{2}}$

c) $\tan 135^\circ = \boxed{-1}$

d) $\sin 270^\circ = \boxed{-1}$

e) $\cos \frac{5\pi}{4} = \boxed{-\frac{\sqrt{2}}{2}}$

f) $\sin \frac{4\pi}{3} = \boxed{-\frac{\sqrt{3}}{2}}$

g) $\tan \frac{5\pi}{3} = \boxed{-\sqrt{3}}$

h) $\cos \frac{9\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

⑦ Convert from radian measure to degrees:

$$\frac{7\cancel{\pi}}{\cancel{3}_{1}}\left(\frac{\cancel{180}^{60}}{\cancel{\pi}}\right) = \boxed{420^\circ}$$

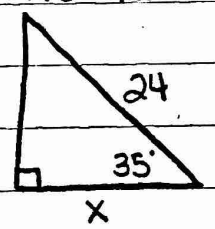
⑧ Convert from degrees to radians:

$$\overset{7}{\cancel{210^\circ}}\left(\frac{\pi}{\underset{6}{\cancel{180}}}\right) = \boxed{\frac{7\pi}{6}}$$

⑨ Find one positive and one negative coterminal angle for an angle with a measure of $\frac{4\pi}{3}$ radians.

Negative: $\boxed{\frac{-2\pi}{3}}$ Positive: $\boxed{\frac{10\pi}{3}}$

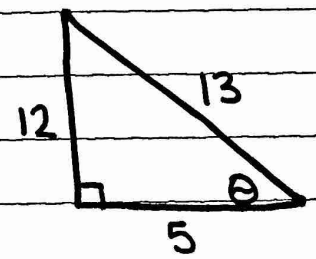
⑩ Solve for x:



$\boxed{x \approx 19.7}$

$\cos 35^\circ = \frac{x}{24}$

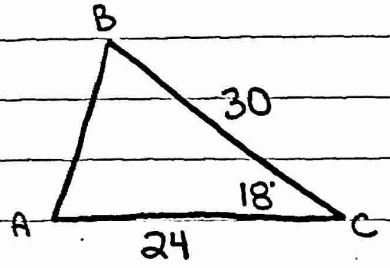
⑪ If $\cos\theta = \frac{5}{13}$ find the EXACT value of $\sin\theta$.



$\sin\theta = \boxed{\frac{12}{13}}$

(12) Solve the following triangles

a)



$c^2 = a^2 + b^2 - 2ab(\cos C)$

$c^2 = 30^2 + 24^2 - 2(30)(24)(\cos 18^\circ)$

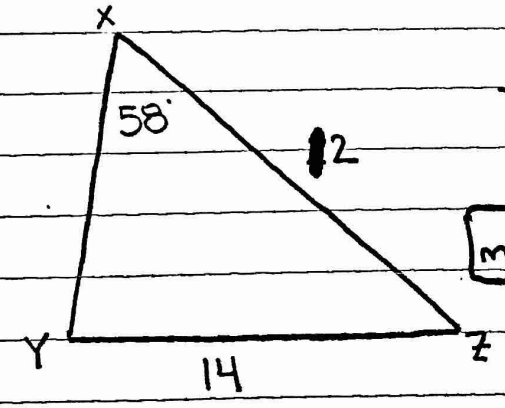
$c^2 \approx 900 + 576 - 1369.5214$

$\boxed{C \approx 10.31884764}$

$\frac{10.3188}{\sin 18^\circ} = \frac{24}{\sin B}$

$\boxed{m\measuredangle B \approx 98.1^\circ}$ $\boxed{m\measuredangle A \approx 63.9^\circ}$

b)



$\frac{14}{\sin 58^\circ} = \frac{12}{\sin Y}$

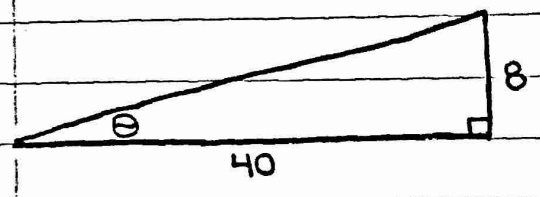
$\boxed{m\measuredangle Y \approx 46.63^\circ}$ or $\cancel{Y \approx 133.37}$

only <u>one</u> triangle!

$\boxed{m\measuredangle Z \approx 75.37^\circ}$

$\boxed{Z \approx 15.97}$

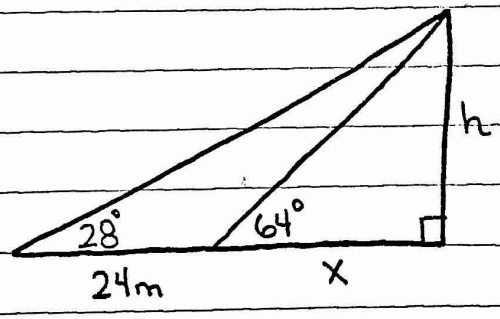
(13) A ramp at a mall has a height of 8 feet and is 40 feet long. What is the angle of elevation for the ramp?



$\tan\theta = \frac{8}{40}$

$\boxed{\theta \approx 11.3^\circ}$

(14) What is the value of "x" in the diagram shown?



$\tan 64^\circ = \frac{h}{x}$

$h = x\tan 64^\circ$

$\tan 28^\circ = \frac{h}{x+24}$

$h = x\tan 28^\circ + 24\tan 28^\circ$

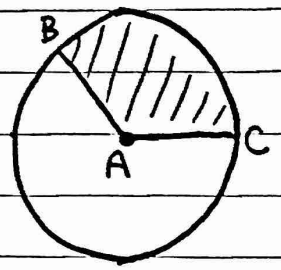
$x\tan 64^\circ = x\tan 28^\circ + 24\tan 28^\circ$

$x(\tan 64^\circ - \tan 28^\circ) = 24\tan 28$ $\quad x = \frac{24\tan 28^\circ}{(\tan 64^\circ - \tan 28^\circ)} \approx \boxed{8.4}$

• What is the value of "h" in the diagram above?

$h = x\tan 64^\circ$ $\quad \boxed{h \approx 17.2}$

(15) What is the area of the shaded sector given $m\angle BAC = 120^\circ$ and radius $AC = 7$ ft.



$$A = \frac{1}{2}(7^2)\left(\frac{2\pi}{3}\right)$$

$$A = \frac{49\pi}{3} \approx \boxed{51.3 \text{ ft}^2}$$

(16) Prove the equation is true by using mathematical induction.

$$2+4+6+8+\ldots\ldots+2n = n(n+1)$$

$n=1$ $\quad 2 = 1(1+1) = 2$ ✓

$n=K$ $\quad [2+4+6+8+\ldots\ldots+2K] = K(K+1)$ ✓

$n=K+1$ $\quad [2+4+6+8+\ldots\ldots+2K] + 2(K+1) = (K+1)(K+2)$

$$K(K+1) + 2(K+1) = (K+1)(K+2)$$

$$(K+1)(K+2) = (K+1)(K+2)$$

(17) Prove the equation is true using mathematical induction.

$$1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$n=1$ $\quad 1^2 = \frac{1(1+1)(2+1)}{6} = \frac{6}{6} = 1$ ✓

$n=k$ $\quad [1^2+2^2+3^2+\ldots+k^2] = \frac{k(k+1)(2k+1)}{6}$

$n=k+1$ $\quad [1^2+2^2+3^2+\ldots+k^2]+(k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$

$$\frac{k(k+1)(2k+1)}{6} + \frac{6(k+1)^2}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$\frac{(k+1)[k(2k+1)+6(k+1)]}{6} = \quad " \quad "$$

$$\frac{(k+1)(2k^2+k+6k+6)}{6} = \quad " \quad "$$

$$\frac{(k+1)(2k^2+7k+6)}{6} = \quad " \quad "$$

$$\frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} \checkmark$$

Extra Credit:

① What is the one word that when you say it, it is no longer there?

② What is the first whole # starting from zero that you need the letter "a" to spell it?

③ What can you put in a box and make it weigh less? (NOT A GAS)